E203. Resonance in RLC circuits

(non-radiative dipole coupling)

1. Introduction

The resonance phenomenon is based on stimulation of the vibration whose amplitude may be disproportionately high in relation to the power which induce this vibrations. Electric resonance occurs when the frequency of the oscillating system is close to or equal to the frequency of the element forcing vibration. Electrical oscillation circuit consists of a capacitor, the coil and the ohmic resistance associated with the coil. Ohmic resistance absorbs electromagnetic vibration energy thus vibrations are damped and fading. To avoid damping of oscillations (leading to the disappearance of the energy in the circuit), an additional energy has to be brought to the system which compensates the loss by heat and radiation. Oscillations in electric circuit are based on the formation and disappearance of the electric field in the capacitor and the magnetic field in the coil.

Oscillatory circuit is a source of electrical and magnetic disturbances that propagate through space as an electromagnetic wave. The easiest way to deliver the power to the oscillator (called resonator) is its coupling with another circuit, called generator. Inductive coupling by means of mutual induction of the two coils is frequently used. The coupling coefficient *k* is dependent on the inductance of the generator coil L_1 and resonator coil L_2:

$$k = \frac{L_{12}}{L_1 \, L_2}, \tag{1}$$

where L_{12} is the mutual inductance of the coils L_1 and L_2. For strong coupling two maxima for different frequencies are observed, while for weak coupling both resonance frequencies are almost equal and only one maximum is observed (Figure 1).

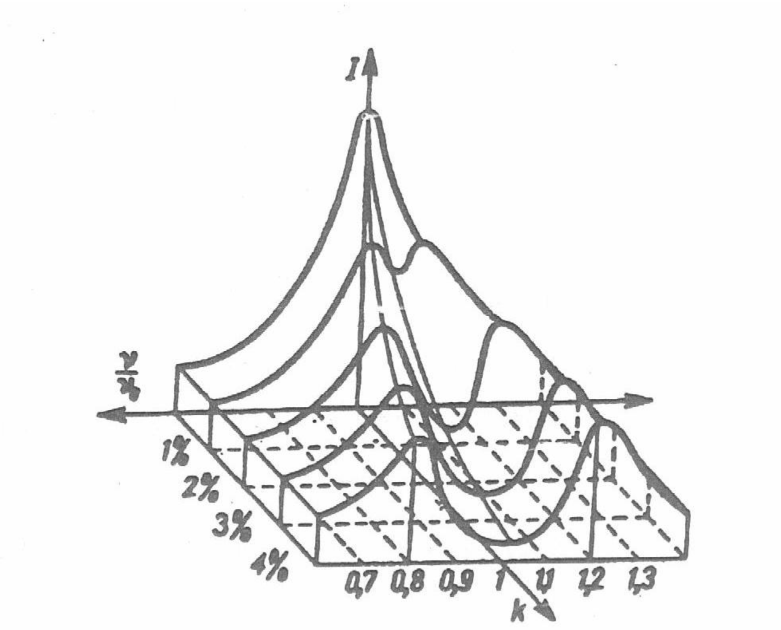

Figure 1. The shape of the resonance curves depending on the coupling coefficient *k*.

In this laboratory exercise the resonator is parallel RLC circuit (Figure 2) in which the resistor *R* can be varied. The resultant resistance R_w is equal:

$$R_W = \frac{R\,r}{R + r}, \tag{2}$$

where *r* is internal ohmic resistance of the coil.

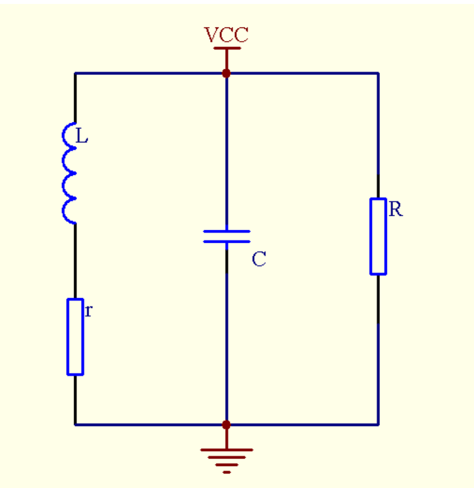

Figure 2. Parallel RLC circuit of the resonator.

The maximum of the resonance occurs for the frequency f_0 equal to:

$$f_0 = \frac{\sqrt{\frac{1}{LC} + \left(\frac{R_W}{L}\right)^2}}{2\pi}, \tag{3}$$

which for small damping simplifies to $f_0 = 1/(2\pi\sqrt{LC})$.

Q-factor is the parameter characterizing the power dissipation of the circuit. It is defined as the ratio of the vibrational energy at any time to the energy lost in one period. For the parallel RLC circuit it is equal to:

$$Q = \frac{R_W}{2\pi\ f_0\ L}. \tag{4}$$

Q-factor can be also determined from the resonance curve:

$$Q = \frac{f_0}{f_2 - f_1}, \tag{5}$$

where f_1 is the lower half-power frequency and f_2 is the upper half-power frequency (the frequencies at which the power passed through the circuit has fallen to half the value passed at resonance).

For small coupling the generator coil can be considered as magnetic dipole. Then the electromagnetic field produced by such element decreases with the distance *d* as $1/d^3$.

2. Apparatus and measurements

The setup consist of generator and resonator (two parallel RLC circuits), in which the distance d between two coils L_1 and L_2 can be varied. Increasing the distance makes the coupling between the coils weaker. A digital generator (after turning it on, the saved settings SET01 should be recalled) is used to apply different frequency to the generator circuit and to polarize (constant voltage) the resonator circuit to tune the resonance frequency by changing the capacitance. In the resonator the ohmic resistance R can be changed. The integrated voltage is measured by the digital oscilloscope (in channel 1).

The experiment is controlled by the application Obwody RLC (Figure 3). After setting the initial (Częstotliwość początkowa) and final (Częstotliwość końcowa) frequencies in MHz, as well as number of points to be measured (Liczba punktów częstotliwości), the green button starts the acquisition. When the resonance curve (amplitude in volts versus frequency in MHz, see Figure 3) is measured, it should be saved in the text file by pressing the Zapisz dane button. Then, the data can be analyzed in the external program (e.g. Excel) to determine the following parameters: resonance frequency (in MHz), maximum amplitude of the oscillation (in volts) and Q-factor (from eq. (5) and after raising the amplitude to the square to obtain the power).

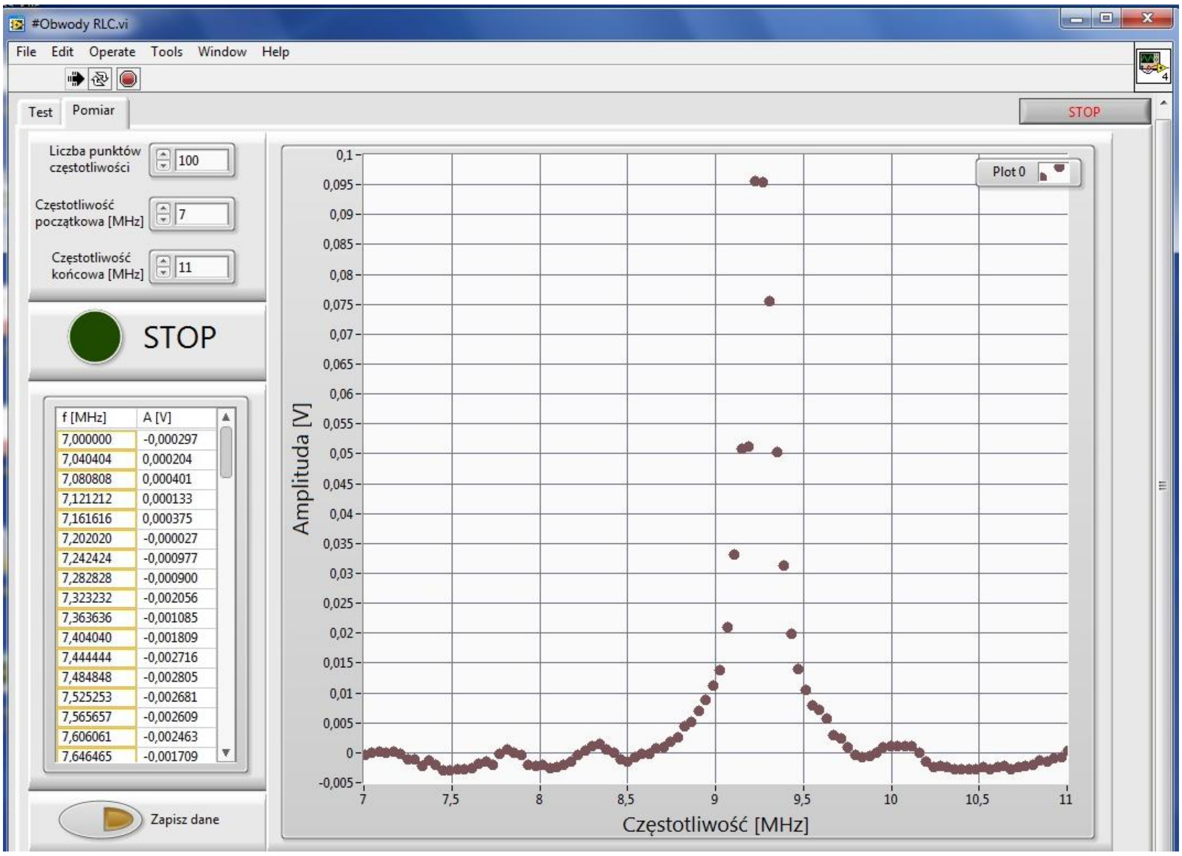

Figure 3. Screenshot of the application used to measure the resonance curve.

The maximum amplitude can be plotted as a function of $1/d^3$ (Figure 4). For sufficiently large distances the dependence is linear (Figure 5), which means that the magnetic dipole approximation (weak coupling) is valid. The critical distance from which the linear dependence is present can be established and compared to the diameter of the coil.

Next, the dependence of the amplitude and Q-factor on the ohmic resistance R in the resonator can be determined and discussed, comparing with formula (4).

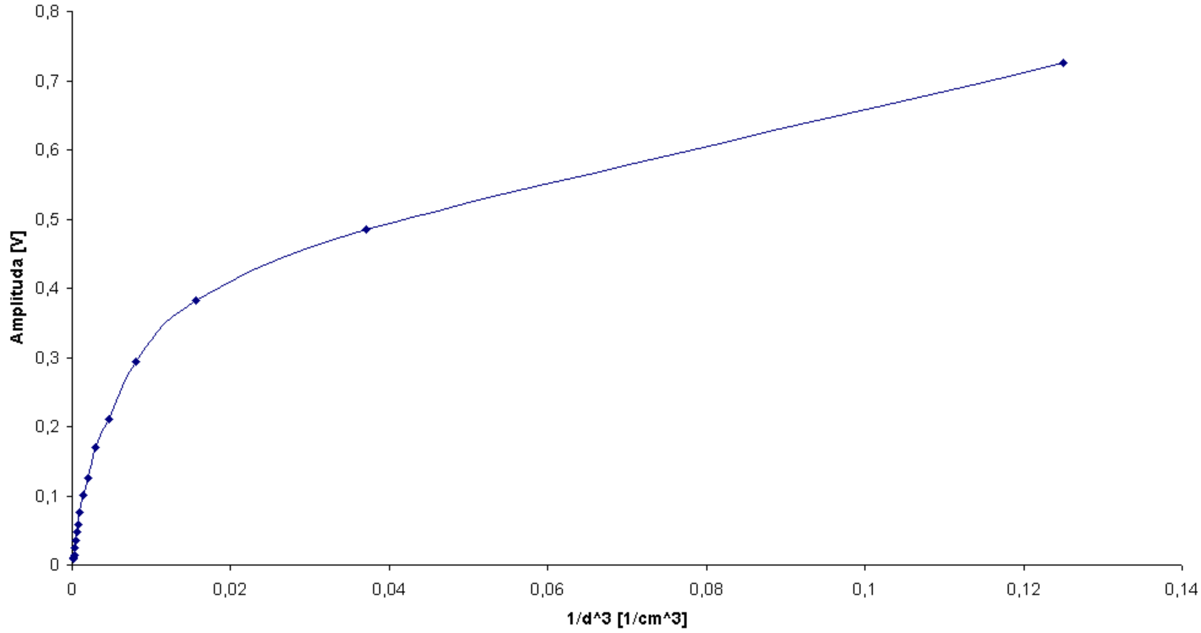

Figure 4. Example dependence of the amplitude on the distance between the coils ($1/d^3$).

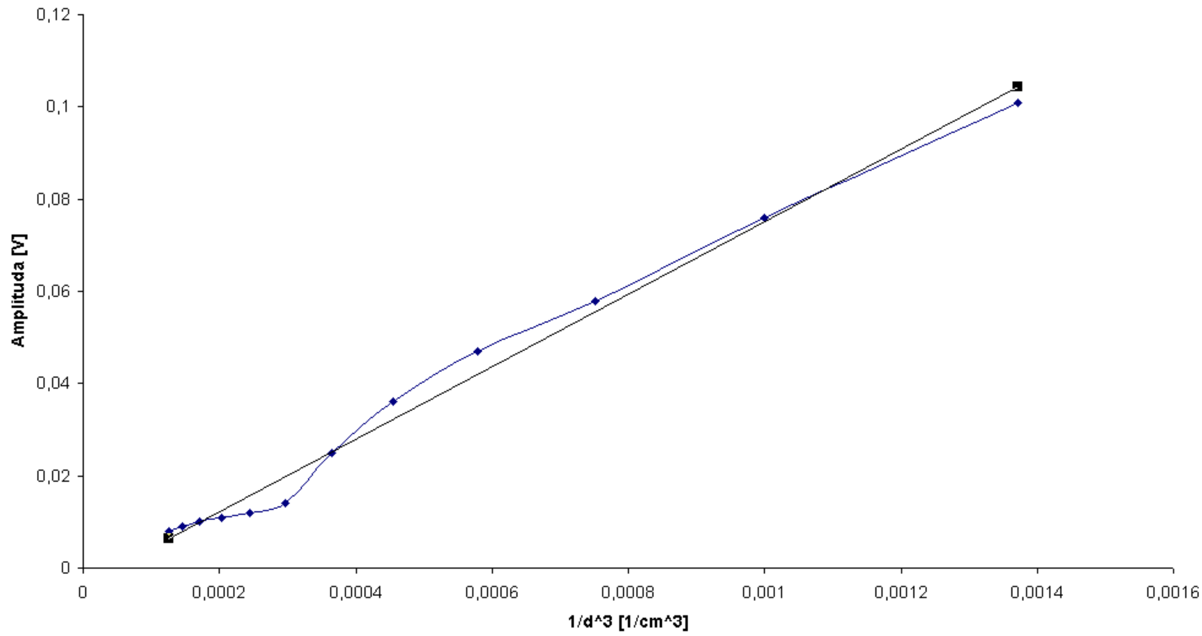

Figure 5. Example of the linear dependence of the resonance amplitude on the value $1/d^3$ for large distances d between the coils. The linear regression can be used to confirm the linear relationship.