D101. Properties of sound waves;

Speed of sound and beats

Aim: *To write a measuring application for investigation of selected properties of sound waves:*
- *measurement of speed of sound on the basis of the time of acoustic pulse propagation*
- *investigation of the wave interference (beats)*

Problems*:*
- *The wave equation, interrelations between the frequency f, angular frequency ω, wavelength λ, wave velocity v, period T, wavenumber k, energy of acoustic wave, Fourier analysis.*
- *The formula for the sum of sines: sin(α+β).*

1. Introduction

The equation describing the plane wave of frequency f and wavelength λ, propagating in the direction x takes the form:

$$y = A \sin(\omega t - kx), \tag{1}$$

where y is the deviation from the equilibrium position, A is the amplitude of deviation, $\omega = 2\pi f$ is the angular frequency, k is the wavenumber ($k = 2\pi/\lambda$). In reference to the sound wave, y has a sense of a change in acoustic pressure. In this experiment, the sources of sound are two column loudspeakers connected to the computer sound card of a computer. Acoustic signal is recorded via a usual microphone connected to the same sound card. As the sound wave is plane it does not undergo polarization.

Placing a microphone at a certain position (x = constant), we get an acoustic signal in the sinusoidal form:

$$y = A \sin(\omega t - \varphi), \tag{2}$$

where $\varphi = kx$.

Tasks:
1. Write a program for generation of tones, based on the sound card and a set of loudspeakers. The frequencies and amplitudes should be controlled independently for the left and right channel.
2. Write a program for recording of sound by a microphone and sound card.
3. Record exemplary courses of acoustic waves coming from one source. Optimize the mutual positions of the loudspeakers and the microphone and optimize the settings in Properties of recording in the operation system. .
4. Comment: details on the sound generating and recording functions can be found in the section "Technical comments" given below. Start with tone generation, if you are satisfied with the result, add the function of recording. Both loops responsible for generation and recording of sound can be found in the same block diagram.

2. Determination of sound speed from measurements of acoustic pulse propagation time

The speed of sound can be determined by a more direct method. Assuming that sound propagates in air at the same speed, and making use of the definition of velocity in a uniform motion $v=ds/dt$, the speed of sound v can be found by generation of a short acoustic pulse (wave packet) and measurement of the time dt needed by the pulse to cover the distance ds
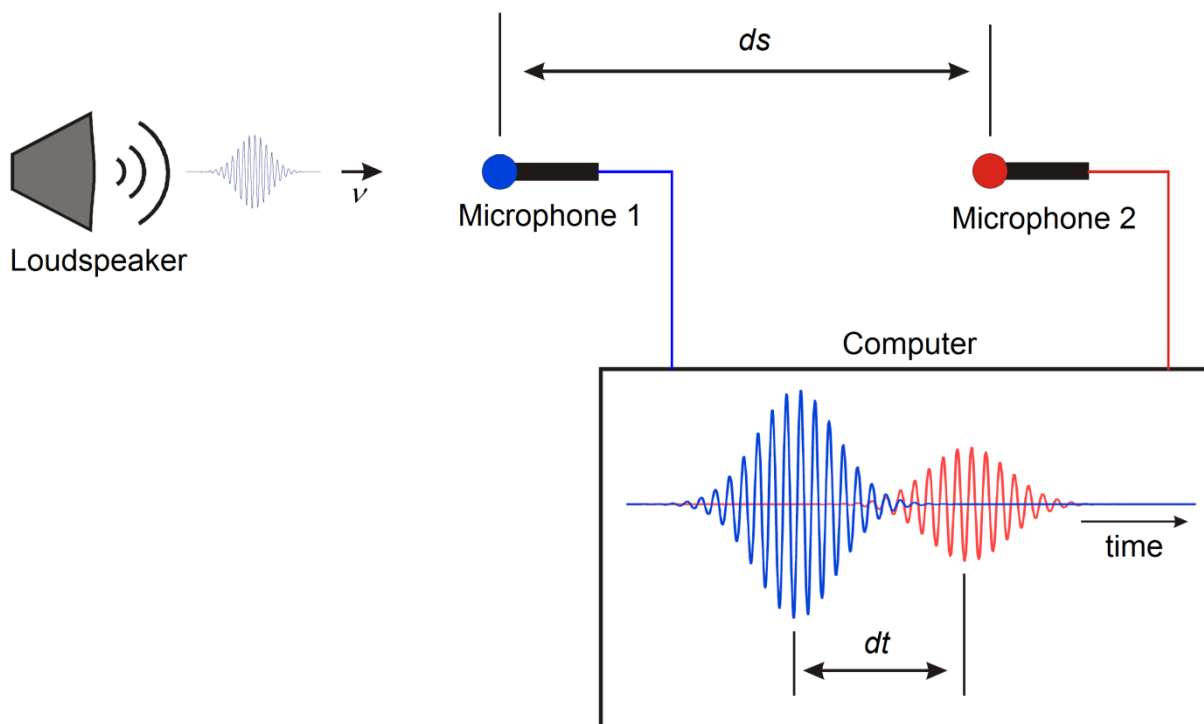
A wave packet can be generated by multiplication of the sinusoidal wave by the bell-type function, e.g. a Gauss function, $\exp[-((t-t_0)/\Delta t)^2]$, where t_0 is the position of the center and Δt is the width of the bell.

Tasks
1. Use one loudspeaker and two microphones set apart at a distance ds.
2. Modify the generator so that it would generate a tone of a given frequency whose amplitude is modulated by a Gauss function. In this way a pulse is obtained. The pulse duration is described by Δt (half width or full width at half maximum (FWHM)).
3. Record the acoustic signal from two microphones set at a distance ds. The image recorded should be made of two pulses whose maxima are shifted in time by dt. Use the function Trigger and Gate.vi to obtain a stable image of the pulses. Using the cursors, measure the time of pulse propagation dt in the Waveform Graph.
4. Measure the time of pulse propagation for a few distances between the microphones.   Calculate the speed of sound from analysis of regression of the linear dependence ds(dt).

3. Investigation of wave interference

Waves coming from different sources, reaching simultaneously the same point is space undergo interference, which means that their momentary deviations. For two waves propagating in one direction, non-polarized or polarized in the same plane, we can write

$$y = A_1\sin(\omega_1 t - k_1 x) + A_2\sin(\omega_2 t - k_2 x) \tag{6}$$

If the frequencies of two interfering waves differ slightly, the phenomenon of beating takes place. This phenomenon heard as a modulation of the sound amplitude with the frequency equal to the difference between the frequencies of interfering waves. Neglecting the spatial term kx, we get:

$$y = A[\sin(\omega_1 t) + \sin(\omega_2 t)] = 2A\cos[\tfrac{1}{2}(\omega_1 - \omega_2)t]\ \sin[\tfrac{1}{2}(\omega_1 + \omega_2)t], \tag{7}$$

in which the slowly oscillating term $\cos[\tfrac{1}{2}(\omega_1 - \omega_2)t]$ can be treated as modulation of the amplitude of faster oscillations of the angular frequency $\tfrac{1}{2}(\omega_1 + \omega_2)$.

Beats

Time
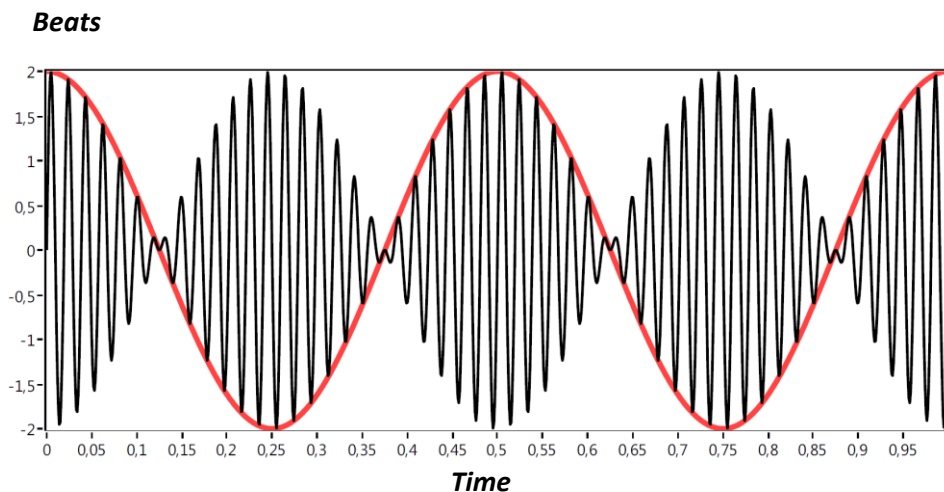
The depth of modulation decreases if the interfering waves differ in amplitude.

Beats

Time
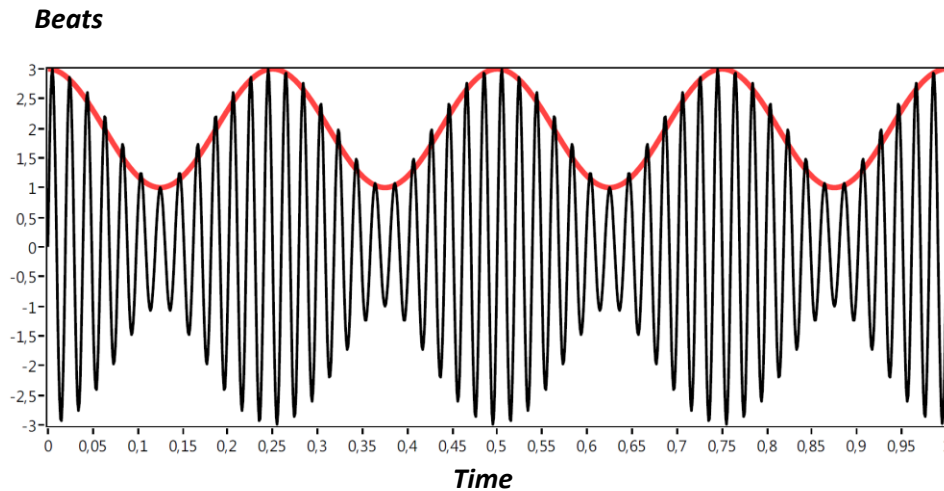
Although it is difficult to describe analytically the envelope of such beats, we can write the following equations:

$$y = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$
$$= A_1 \sin(\omega_1 t) + A_1 \sin(\omega_2 t) + (A_2 - A_1) \sin(\omega_2 t)$$
$$= 2A_1 \cos[\tfrac{1}{2}(\omega_1 - \omega_2)t]\, \sin[\tfrac{1}{2}(\omega_1 + \omega_2)t] + (A_2 - A_1)\sin(\omega_2 t). \tag{8}$$

As follows, the interference of oscillations of similar frequencies can be expressed as deeply modulated oscillations of an intermediate frequency and oscillations of the component with highest amplitude. Unfortunately, this description does not fit the reality as the difference between the frequencies of these oscillations causes smoothing of the sharp fragments so that the envelope resembles a sinusoidal function of the frequency equal to the difference $(\omega_1 - \omega_2)$, and not to half of this difference.

Working on this

experiment we noted that the envelope of beats is rather well described by a combination of the two oscillating functions:

$$y = a\cos[\tfrac{1}{2}(\omega_1 + \omega_2)t](b\cos[(\omega_1 - \omega_2)t] + c) \tag{9}$$

We leave the explanation of this observation to the inquiring students. In this experiment, you can determine the dependencies of parameters a, b, c on the ratio of amplitudes of the signals A_1/A_2. The values of these parameters you can get from manual fitting, e.g. by using slides in LabView.

Please note that beating is observed only when the waves can interfere on a detector. They are bound to occur if the source of sound are loudspeakers and a small microphone is a detector. For

comparison, try to listen to the same sound through the headphones (different frequencies in the left and right channels) or after shifting the loudspeakers directly to your ears (left to the left ear and right to the right one).

Task
Place two column loudspeakers next to each other and a microphone in front of them, at a distance of 20-40 cm. Generate the sound of slightly different frequencies in both loudspeakers. Measure the signal from the microphone for different combinations of frequencies and amplitudes of the signals from both loudspeakers. Describe the obtained envelope by equation (7) or (9). If you describe it by equation (9)try to interpret the parameters a, b, c in terms of the amplitudes of the component waves. The parameters of the component waves can be determined independently, by switching off individual columns.

References

1. Henryk Szydłowski " Pracownia Fizyczna", Wyd. Nauk. PWN, Warszawa1994.

2. David Halliday, Robert Resnick and Jearl Walker "Podstawy fizyki" Wyd. Nauk. PWN, Warszawa 2003.

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Technical comments:

1. Because of the necessity of double channel (stereo) measurements in some measurements the microphones can be connected through the amplifier to the linear input of the sound card "line in" so that one of them works on the left channel, while the other with the right one.   For the tasks for which one microphone is sufficient, it can be connected to the microphone input, checking the settings "Właściwości nagrywania" (Properties of recording), see point 2.

2. Check the settings "Właściwości nagrywania" (Properties of recording) in the system. Click twice the loudspeaker icon in the Windows tool bar. In the menu Options choose Properties and mark the volume of recording. Make sure that the box at linear input or microphone is marked, as needed. Set with a slide the volume of recording at least at the middle of the slide range.

3. A few comments on sound generation with the sound card

- The functions permitting sound generation are to be chosen from the following Functions/Graphics & Sound/Sound/Sound Output LabView palette,
- To generate a tone, it is preferable to use the function Sine Waveform.vi. Apart from the frequency and amplitude of sinus, the function needs the specification of the sampling rate and the number of samples of which the generated signal is to be composed. These parameters are needed to generate the waveform type data in LabView. It is a special data type which contains besides the samples table also information on the digital-to-analog converter setup. When using a music card, the sampling rate should be set to 44100 samples/second.
- The diagram of the program given below generates a monophonic tone of the pre-set amplitude and frequency. For some experiments, e.g. investigation of beats, it is necessary to generate a stereo signal. To do it, introduce the appropriate change in the cluster Sound Format of the function Sound Output Configure. Use two functions Sine Waveform.vi. Then build the array from both waveforms (function Build Array) and send it to the function generating the Sound Output Write.vi.
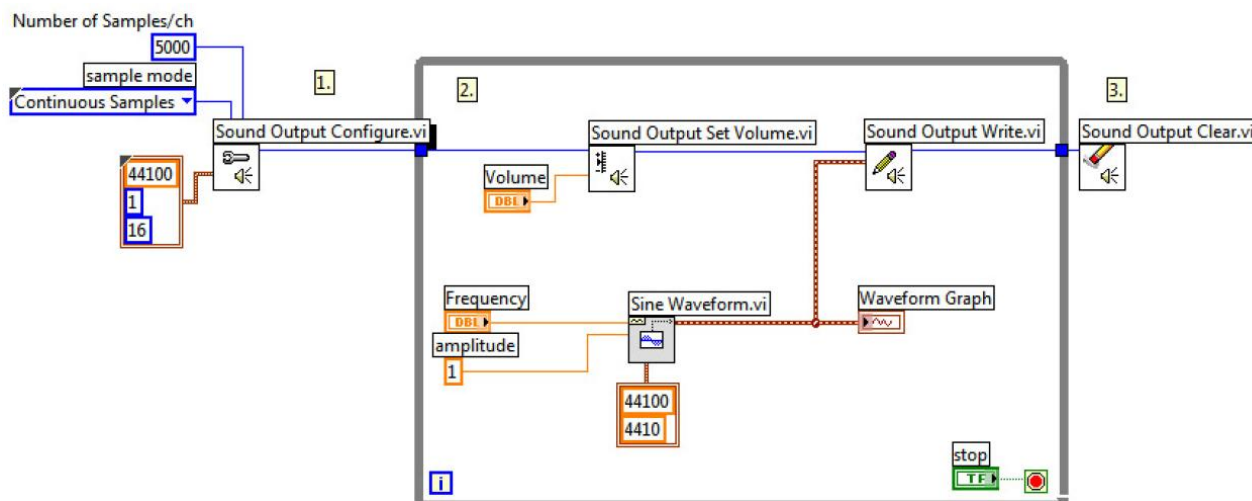
1. Configure the device in continuous generation mode.
2. Repeat until user press Stop:
- generate the sinus function (as a waveform) at set frequency and amplitude = 1. Show the data on the graph,
- set the signal volume at the required level (0 - 100%),
- play the waveform on the device by writing the data into the sound card.
3. Stop sound generation and clear all resources

4. Comments on sound recording with the use of sound card

- The functions permitting sound generation are included in the following set: Functions/Graphics & Sound/Sound/Sound Input LabView palette,

- The diagram of the program given below generates the stereo signal and displays the data in a graph. The value written in the box *Number of Samples/ch* in the function *Sound Input Configure.vi* determines the number of samples of the signal so the time length of the signal recorded by *Sound Input Read.vi*. When you increase this value, the time length of signal recording will extend.
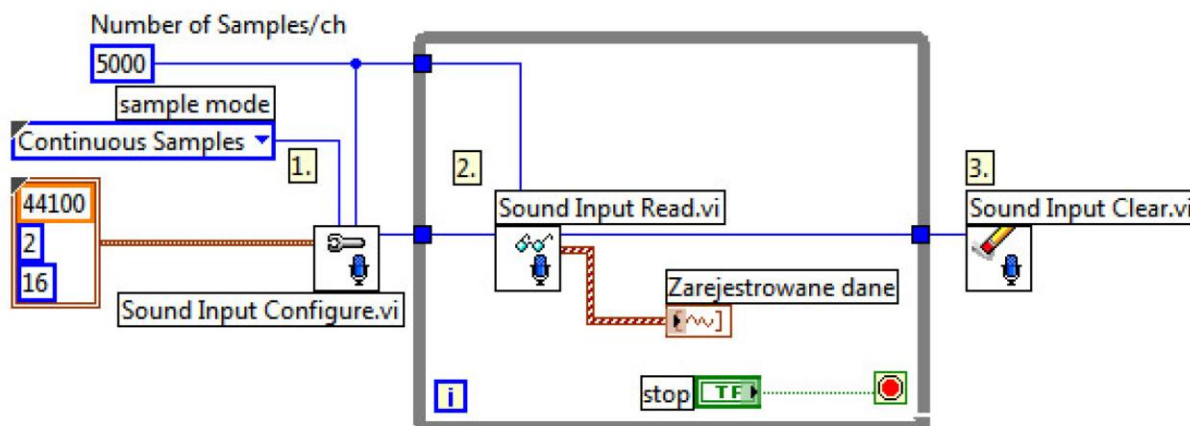
1. Configure the device in continuous registration mode.
2. Collect samples and show them in Graph until user press Stop.
3. Stop registration and clear all resources.

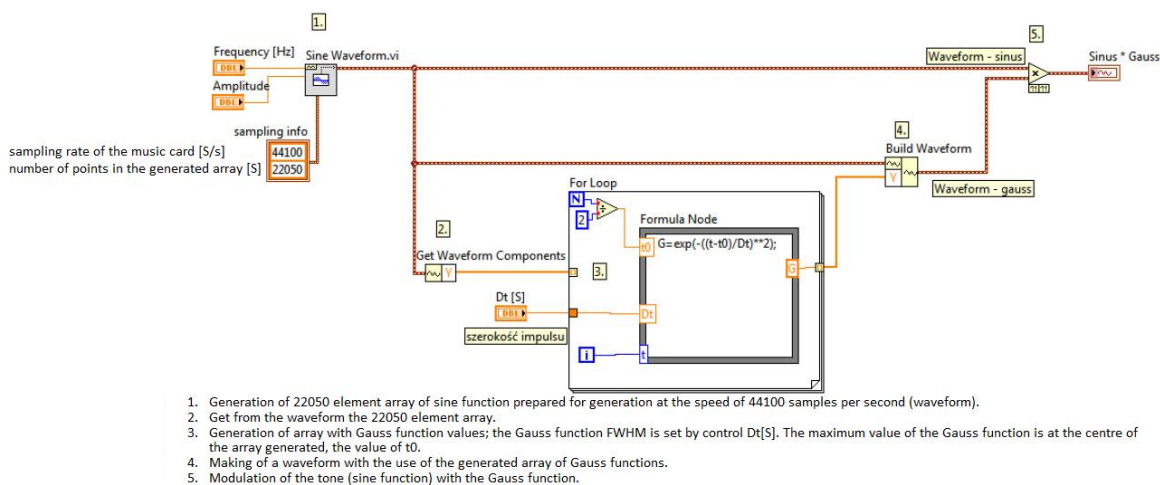5. Modulation of tone amplitude; formation of a wavepacket.

We are interested in generation of the function $y(t) = A(t)T(t)$ where $A(t)$ is the amplitude varying in time (Gauss function) and $T(t)$ is the sine so the tone:

$$A(t) = e^{-[(t-t_0)/\varDelta t]^2}$$
$$T(t) = \sin(2\pi f t)$$

To get properly generated modulated function, it must by expressed as a *Waveform*. Start with using the function *Sine Waveform.vi*. Use the function *Get Waveform Components* to collect from the waveform only the array of function values. Then, make the array of the values of Gauss function of the same size as the array of sines (loop *FOR* + structure of *Formula Node*). The array obtained use for building a waveform by using *Build Waveform*. The two waveforms for the Gauss function and for sinus function can be multiplied. The resultant waveform can be sent to the sound generating function *Sound Output Write.vi*.

The diagram of an exemplary VI realising the modulation of the sine function amplitude with a Gauss function is shown below.

1. Generation of 22050 element array of sine function prepared for generation at the speed of 44100 samples per second (waveform).
2. Get from the waveform the 22050 element array.
3. Generation of array with Gauss function values; the Gauss function FWHM is set by control Dt[S]. The maximum value of the Gauss function is at the centre of the array generated, the value of t0.
4. Making of a waveform with the use of the generated array of Gauss functions.
5. Modulation of the tone (sine function) with the Gauss function.

6. When fitting a curve to the beats signal make use of the possibility of changing parameters *a*, *b*, *c* by slide controls or similar controls, as it permits fast choice of correct values of these parameters. Try to generate a complete function with a fast oscillating term, by setting its frequency to the value obtained from the measurements and by changing only the amplitude of this fast component.